Work

The bridge between force and energy.
Work is a scalar.
$W = F \Delta x \cos \theta$
F: force (N)
Δx : displacement (m)
θ: angle between force and displacement (usually 0°)
SI Unit: Joule (N m)

Counterintuitive Results

There is no work if there is no displacement.
Forces perpendicular to displacement don't work.
By doing positive work on an object, a force or collection of forces increases its **mechanical energy** in some way.
The two forms of mechanical energy are called **potential** and **kinetic energy.**

Problem: Work (B-1988)

6. A horizontal force F is used to pull a 5-kilogran block across a floor at a constant speed of 3 meters per second. The frictional force between the block and the floor is 10 newtons. The work done by the force F in 1 minute is most nearly 60s

(A) 0 J (B) 30 J (C) 600 J
(D) 1,350 J (E) 1,800 J

F_{net}

Show your work:

$W_{net} = F_{net} d_{net}$

$V = 3$ m/s
$m = 5$ kg
$f_f = 10$ N
$t = 60$s $a = \frac{v}{t} = \frac{3 \text{ m/s}}{60\text{s}} = .05 \text{ m/s}^2$

Kinetic Energy

Energy due to motion
$K = \frac{1}{2} m v^2$
K: Kinetic Energy in Joules.
m: mass in kg
v: speed in m/s

Problem: Kinetic Energy (B-1988)

3. Which of the following quantities is a scalar that is always positive or zero?

(A) Power (B) Work (C) Kinetic energy
(D) Linear momentum (F) Annular momentum

State your reasoning:

The Work-Energy Theorem

$W_{net} = \Delta K$
Net work is used in this theorem. This is work due to ALL FORCES acting upon object.
When net work is positive, the kinetic energy of the object will increase (it will speed up).
When net work is negative, the kinetic energy of the object will decrease (it will slow down).
When there is no net work, the kinetic energy is unchanged (constant speed).

Work and graphs

The area under the curve of a graph of force *vs* displacement gives the work done by the force in performing the displacement.
Springs: stretching
Springs:compressing

Power

The *rate* of which work is done.
When we run upstairs, t is small so P is big.
When we walk upstairs, t is large so P is small.
$P = W/t$
work/time
$P = F V$
(force)(velocity)
SI unit for Power is the Watt.
1 Watt = 1 Joule/s

Problem: Power (B-1998)

2. A student weighing 700 N climbs at constant speed to the top of an 8 vertical rope in 10 s. The average power expended by the student to overcome gravity is most nearly 8m

(A) 1.1 W
(B) 87.5 W
(C) 560 W
(D) 875 W
(E) 5,600 W

Show your work:

$P = \frac{Fd}{t} = \frac{700 \cdot 8}{10} = 560$

How We Buy Energy…

The kilowatt-hour is a commonly used unit by the electrical power company.
Power companies charge you by the kilowatt-hour (kWh), but this not power, it is really energy consumed.
1 kW = 1000 W
1 h = 3600 s
1 kWh = 1000J/s • 3600s = 3.6×10^6J

Problem: Power (B-1998)

5. Units of power include which of the following?

   I. Watt
   II. Joule per second
   III. Kilowatt-hour

   (A) I only
   (B) III only
   (C) I and II only
   (D) II and III only
   (E) I, II, and III

State your reasoning:

Problem: Power (B-1998)

9. A child pushes horizontally on a box of mass m which moves with constant speed v across a horizontal floor. The coefficient of friction between the box and the floor is μ. At what rate does the child do work on the box?

   (A) μmgv
   (B) mgv
   (C) $v/\mu mg$
   (D) $\mu mg/v$
   (E) μmv^2

Show your work:

Force Types

Conservative forces:

Work in moving an object is ***path independent***.
Work in moving an object along a closed path is zero.
Work done against conservative forces increases potential energy; work done by them decreases it.
Ex: gravity, springs

Non-conservative forces:

Work is ***path dependent***.
Work along a closed path is NOT zero.
Work may be related to a change in total energy (including thermal energy).
Ex: friction, drag

Potential energy

Energy an object possesses by virtue of its position or configuration.
Examples:
Gravitational Potential Energy
Spring Potential Energy

Potential energy is related to work done by CONSERVATIVE FORCES only.

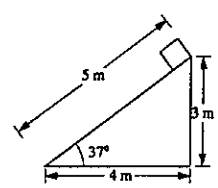
$\Delta U_g = -W_g$ (gravity)
$\Delta U_g = -W_s$ (spring)

Gravitational potential energy close to earth's surface.

$W_g = -mgh$ (close to earth's surface)
m: mass of object (kg)
g: acceleration due to gravity (m/s^2)
h: height change (m) (positive to go up)
$\Delta U = -W_g = mgh$

*Note: we calculate **changes** in potential energy only using this method. We assign the potential energy to be zero at some certain point, usually the surface of the earth.*

Problem: Work due to gravity (B-1993)



A plane 5 meters in length is inclined at an angle of 37°, as shown above. A block of weight 20 newtons is placed at the top of the plane and allowed to slide down.

63. The work done on the block by the gravitational force during the 5-meter slide down the plane is most nearly

   (A) 20 J (B) 60 J (C) 80
   (D) 100 J (E) 130 J

Show your work:

ADVANCED TOPIC

Gravitational potential energy changes far from earth's surface.

$U_g = -GM_e m/r$ (close to earth's surface)

- G: Universal gravitational constant
- M_e: Mass of earth
- m: mass of another object
- r: distance from center of earth

U_g has been defined to be zero when an object is infinitely far from the earth, and it gets increasingly negative as an object approaches the earth.

Note: This literal definition is impractical in most problems, but this is the equation that must be used to calculate ΔU when you a very far from the earth's surface.

Spring potential energy

Springs also can possess potential energy (U_s).

$U_s = ½ kx^2$

U_s: spring potential energy (J).

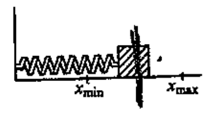
k: force constant of a spring (N/m).

x: the amount the spring has been stretched or compressed from its equilibrium position (m).

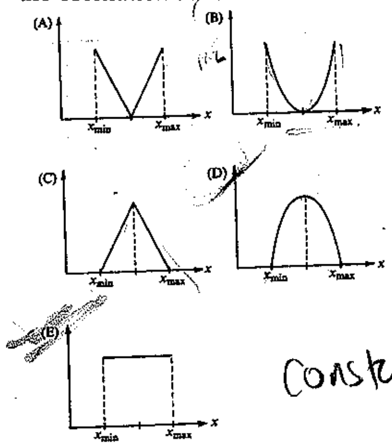
U_s is zero when a spring is in its preferred, or equilibrium, position where the spring is neither compressed or extended.

Problem: Conservation of Energy and Springs(B-1988)

Questions 11-12



A block oscillates without friction on the end of a spring as shown above. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max} The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.



11. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?

(A) A (B) B (C) C
(D) D (E) E

Show your work:

12. Which graph can represent the kinetic energy of the block as a function of x ?

(A) A (B) B (C) C
(D) D (E) E

State your reasoning:

Law of Conservation of Mechanical Energy

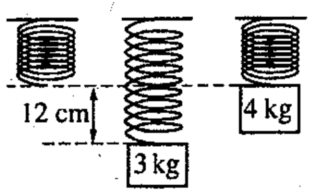
U + K = Constant

ΔU + ΔK = 0

ΔU = -ΔK

Note: Sometimes conservation of energy problems can be best worked with potential energy alone, such as in the problem below.

Problem: Conservation of Energy (B-1998)



38. A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown above. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown above, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?

(A) 9 cm
(B) 18 cm
(C) 24 cm
(D) 32 cm
(E) 48 cm

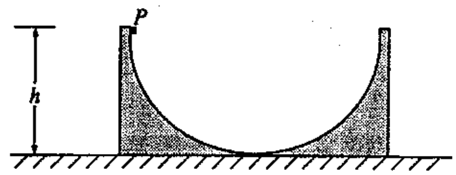
Show your work:

X = .12m
M = 3.0kg

F = 2KX

$\frac{Mg}{X} = K$ $\frac{3 \cdot 10}{.12} = 250 \frac{N}{M}$

Problem: Conservation of Energy (B-1993)



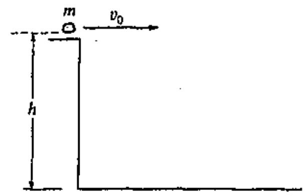
4. The figure above shows a rough semicircular track whose ends are at a vertical height h. A block placed at point P at one end of the track is released from rest and slides past the bottom of the track. Which of the following is true of the height to which the block rises on the other side of the track?

(A) It is equal to h/2π.
(B) It is equal to h/4.
(C) It is equal to h/2.
(D) It is equal to h.
(E) It is between zero and h; the exact height depends on how much energy is lost to friction.

Show your work:

Problem: Conservation of Energy (B-1998)

Questions 59-60



59. A rock of mass m is thrown horizontally off a building from a height h, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is υ_0.

59. How much time does it take the rock to travel from the edge of the building to the ground?

(A) $\sqrt{h\upsilon_0}$ $\quad$ h/υ_0

(B) $h\upsilon_0/g$ $\quad$ $2h/g$

(C) $\sqrt{2h/g}$

Show your work:

60. What is the kinetic energy of the rock just before it hits the ground?

(A) mgh $\quad$ (B) $\frac{1}{2}m\upsilon_0^2$

(C) $\frac{1}{2}m\upsilon_0^2 - mgh$ $\quad$ (D) $\frac{1}{2}m\upsilon_0^2 + mgh$

(E) $mgh - \frac{1}{2}m\upsilon_0^2$ $\quad$ $mgh + \frac{1}{2}mv_0^2 =$

Show your work:

Problem: Conservation of Energy (B-1993)

47. A block of mass m slides on a horizontal frictionless table with an initial speed v_0. It then compresses a spring of force constant k and is brought to rest. How much is the spring compressed from its natural length?

(A) $\frac{v_0^2}{2g}$

(B) $\frac{mg}{k}$

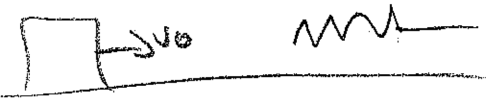
(C) $\frac{m}{k}v_0$

(D) $\sqrt{\frac{m}{k}}v_0$

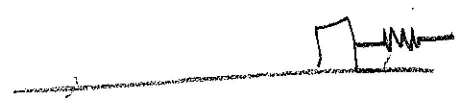
(E) $\sqrt{\frac{k}{m}}v_0$

Show your work:

before



after



Before

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\sqrt{\frac{mv_0^2}{k}} = x$

$\sqrt{\frac{m}{k}}\, v_0 = x$

Conservation of Energy and Dissipative Forces.

Dissipative forces cause loss of mechanical energy by producing heat.

$U + K + E_{int} = \text{Constant}$

$\Delta U + \Delta K + \Delta E_{int} = 0$

The work due to non-conservative forces is responsible for changes in the internal energy.

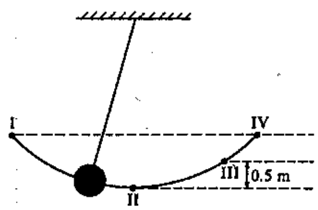
$\Delta E_{int} = -W_{nc}$

Example: kinetic friction always does negative work, since the friction force points in the opposite direction as the displacement.

$W_{nc} = \Delta U + \Delta K$

Problem: Conservation of Energy and Pendulums (B-1988)

Questions 51-52



A ball swings freely back and forth in an arc from point I to point IV, as shown above. Point II is the lowest point in the path, III is located 0.5 meter above II, and IV is 1 meter above II. Air resistance is negligible

51. If the potential energy is zero at point II, where will the kinetic and potential energies of the ball be equal?

(A) At point II
(B) At some point between II and III
(C) At point III
(D) At some point between III and IV
(E) At point IV

State your reasoning:

52. The speed of the ball at point II is most nearly
(A) 3.0 m/s
(B) 4.5 m/s
(C) 9.8 m/s
(D) 14 m/s
(E) 20 m/s

Show your work:

$mgh = \frac{1}{2}mv^2$

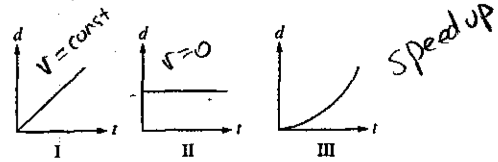
$v = \sqrt{2gh} \quad \sqrt{2 \cdot 10 \cdot 1}$

Momentum
How hard it is to stop a moving object.
Related to both mass and velocity.
For one particle
  $\mathbf{p} = m\mathbf{v}$
For a system of multiple particles
  $\mathbf{P} = \Sigma \mathbf{p}_i = \Sigma m_i \mathbf{v}_i$
Units: N s or kg m/s
Momentum is a vector!

Problem: Momentum (1998)

Questions 43-44

Three objects can only move along a straight, level path. The graphs below show the position d of each of the objects plotted as a function of time t.



43. The magnitude of the momentum of the object is increasing in which of the cases?

(A) II only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

Explain your reasoning:

Impulse (J)
The product of an external force and time, which results in a change in momentum
$J = F t$
$J = \Delta P$
Units: N s or kg m/s

Problem: Impulse (1984)

56. Two planets have the same size, but different masses, and no atmospheres. Which of the following would be the same for objects with equal mass on the surfaces of the two planets?

I. The rate at which each would fall freely
II. The amount of mass each would balance on an equal-arm balance
III. The amount of momentum each would acquire when given a certain impulse

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Explain your reasoning:

Problem: Impulse (1998)

57. A ball of mass 0.4 kg is initially at rest on the ground. It is kicked and leaves the kicker's foot with a speed of 5.0 m/s in a direction 60° above the horizontal. The magnitude of the impulse imparted by the ball to the foot is most nearly

(A) 1 N·s

(B) $\sqrt{3}$ N·s

(C) 2 N·s

(D) $\frac{2}{\sqrt{3}}$ N·s

(E) 4 N·s

Show your work:

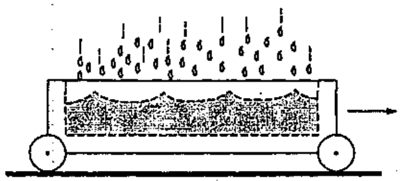
Law of Conservation of Momentum

If the resultant external force on a system is zero, then the momentum of the system will remain constant.

The sum of the momentums before a collision is equal to the sum of the momentums after a collision.

$\Sigma P_b = \Sigma P_a$

Problem: Conservation of Momentum (1998)



4. An open cart on a level surface is rolling without frictional loss through a vertical downpour of rain, as shown above. As the cart rolls, an appreciable amount of rainwater accumulates in the cart. The speed of the cart will

(A) increase because of conservation of momentum
(B) increase because of conservation of mechanical energy
(C) decrease because of conservation of momentum
(D) decrease because of conservation of mechanical energy
(E) remain the same because the raindrops are falling perpendicular to the direction of the cart's motion

Explain your reasoning:

↓ mv
mv = →

Collisions

Follow ***Newton's Third Law*** which tells us that the force exerted by body A on body B in a collision is equal and opposite to the force exerted on body B by body A.

During a collision, external forces are ignored.

The time frame of the collision is very short.

The forces are ***impulsive*** forces (high force, short duration).

Collision Types

Elastic: P is conserved, K is conserved

Inelastic: P is conserved, K is NOT conserved

Perfectly Inelastic means the bodies stick together

Problem: Collisions (1993)

10. Which of the following is true when an object of mass m moving on a horizontal frictionless surface hits and sticks to an object of mass M > m, which is initially at rest on the surface?

(A) The collision is elastic.
(B) All of the initial kinetic energy of the less-massive object is lost.
(C) The momentum of the objects that are stuck together has a smaller magnitude than the initial momentum of the less-massive object.
(D) The speed of the objects that are stuck together will be less than the initial speed of the less-massive object.
(E) The direction of motion of the objects that are stuck together depends on whether the hit is a head-on collision.

Explain your reasoning:

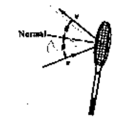
Problem: Collisions (1993)

11. Two objects having the same mass travel toward each other on a flat surface, each with a speed of 10 meter per second relative to the surface. The objects collide head-on and are reported to rebound after the collision, each with a speed of 20 meters per second relative to the surface. Which of the following assessments of this report is most accurate?

(A) Momentum was not conserved, therefore the report is false.
(B) If potential energy was released to the objects during the collision, the report could be true.
(C) If the objects had different masses, the report could be true.
(D) If the surface was inclined, the report could be true.
(E) If there was no friction between the objects and the surface, the report could be true.

Explain your reasoning:

Problem: Momentum Change (1988)



7. A tennis ball of mass m rebounds from a racquet with the same speed v as it had initially, as shown above. The magnitude of the momentum change of the ball is

(A) 0 (B) mv (C) $2mv$
(D) $2mv \sin\theta$ (E) $2mv \cos\theta$

Show your work:

$P_i = -mv\cos\theta$
$P_f = +mv\cos\theta$
$\Delta P = P_f - P_i$
$mv\cos - -mv\cos\theta$
$2mv\cos\theta$

Problem: Collision (1993)

3. A railroad car of mass m is moving at speed V when it collides with a second railroad car of mass M which is at rest. The two cars lock together instantaneously and move along the track. What is the speed of the cars immediately after the collision?

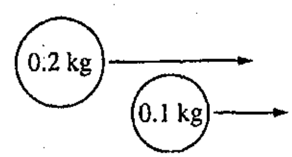
(A) $\frac{v}{2}$
(B) $\frac{mv}{M}$
(C) $\frac{Mv}{m}$
(D) $\frac{(m+M)v}{m}$
(E) $\frac{mv}{m+M}$

$mV = (m_1 + M_2) V_f$

$\frac{mV}{(m_1+M_2)} = V_f$

Show your work:

Problem: Collision (1998)



41. Two objects of mass 0.2 kg and 0.1 kg, respectively, move parallel to the x-axis, as shown above. The 0.2 kg object overtakes and collides with the 0. 1 kg object. Immediately after the collision, the y-component of the velocity of the 0.2 kg object is 1 m/s upward. What is the y-component of the velocity of the 0.1 kg object immediately after the collision?

(A) 2 m/s downward
(B) 0.5 m/s downward
(C) 0 m/s
(D) 0.5 m/s upward
(E) 2 m/s upward

.2kg ↑ 1m/s

Show your work:

.1kg ↓ 2m/s