**Two-dimensional Motion**

**Introduction**

One of Galileo’s greatest insights was that motion in two dimensions may be broken down into horizontal and vertical motion. This process is the *resolution of motion into its components*. Galileo noted, in his early work on kinematics, that an object would continue in motion in a horizontal direction unless and until some force acted upon it. He also noted that gravity would cause an object to undergo uniform acceleration in the downward direction. We investigated the downward motion in the lab exercise on the T2 law. In this lab we couple that uniformly accelerated vertical motion with a horizontal motion at constant velocity and show that the motion is parabolic.

**Theory**

The vertical motion of an object in a uniform gravitational field is described by Equations (1) and (2).

Vy = Voy -gT (1)

y = yo­  + VoyT - (g/2)T2 (2)

In these equations Vy is the object’s velocity in the y-direction at time T. Voy is the object’s initial velocity in the y-direction, y is the position of the object at time T, yo is the initial position of the object in the y-direction, and g is the magnitude of the acceleration of gravity, usually taken to be 9.8 m/s2.

Let us consider the motion of an object starting a distance *H* above the floor and released from rest. The noun “rest” implies that Voy is zero. The initial position of the object is *H*. When the object hits the floor its final position is zero. The time taken for the object to hit the floor is found using Equation (2).

1. = *H* + (0)T - (g/2)T2

T = (2*H*/g)1/2 (3)

If we roll a ball off the top of a table of height *H*, it strikes the ground in a time T given by Equation (3). If the ball has a horizontal velocity Vx as it leaves the table that velocity is unchanged as it falls to the floor. The motion of the object through the air is described by Equation (4).

x = xo­  + VoxT (4)

If the initial horizontal position of the object is zero, and the position when it hits the floor is *R*, Equation (4) may be written

*R* = VoxT . (5)

**Experiment**

We wish to give the ball a reproducible velocity in the experiment so we construct a ball ramp out of two pieces of aluminum channel. The horizontal piece has an internal diameter of ¾”, and the ramp piece has an internal diameter of 5/8”. The ramp is constructed by fitting the 5/8” channel into the ¾” channel about 6 “ from the end of the horizontal piece. Four wooden blocks are used to raise the ramp. The blocks are placed 1 meter from the junction of the two pieces of channel.

Once the ramp is constructed it is taped to the table with painter’s tape as in Figure 1. Care must be taken to not allow the ramp to move in the course of the experiment.

A piece of paper is taped with painter’s tape to the floor below the point where the ball leaves the ramp. A plumb bob is used to mark the point on the paper directly below the point at which the ball leaves the ramp.

The steel ball is released from a point on the ramp above the blocks. The release point should be marked with tape so that it is the same for all trials. The lab group notes the spot where the ball hits the floor, and extends sheets of paper that are taped to the first sheet of paper to the impact point.

A piece of carbon paper centered over the impact point, and another sheet of paper placed on top of it. Neither the carbon paper nore the sheet on top of it are taped to the floor.

The ball is released down the ramp five times, making five marks on the sheets taped to the floor.

**Analysis**

Using Equation (2) and the appropriate measurements, calculate the time T required for the ball to fall to the floor.

Using the time T and the five impact measurements, calculate the velocity of the ball in the x-direction.

Calculation of Vox is 60% of the lab grade.

The remaining 40% are obtained by solving a ballistics problem. The lab group must calculate where to put a circular target that is 30 centimeters above the floor so that the ball goes through the target. Of the remaining 40% of the lab grade, 20% is for the calculation, and 20% for actually getting the ball through the hoop. The lab group has only one trial to get the ball through the hoop.